ANALYZING PERFORMANCE OF GARCH MODELS IN NSE

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Abstract

The study uses three different models GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) to analyze the volatility of Nifty of National Stock Exchange (NSE) of India from January 1, 2010 to July 4, 2014. The results reveal persistence of volatility and the presence of leverage effect implying impact of good and bad news is not same. To evaluate the models, various model selection and forecasting performance criterions like AIC, SBC, RMSE, MAE, MAPE and TIC are employed. Our results indicate that GARCH (1,1) has better forecasting ability in NSE implying impact of good and bad news is not same. To evaluate the models, various model selection and forecasting performance criterions like AIC, SBC, RMSE, MAE, MAPE and TIC are employed. Our results indicate that GARCH (1,1) has better forecasting ability in NSE.

JEL Classification: G14, C32

Key words: Volatility clustering, GARCH, EGARCH, TGARCH, RMSE, MAE, MAPE, TIC


Various features of stock returns have been extensively documented in the literature which are important in modeling stock market volatility. It has been found that stock market volatility is time varying and it also exhibits positive serial correlation (volatility clustering). This implies that changes in volatility are non-random. Moreover, the volatility of returns can be characterized as a long-memory process as it tends to persist (Bollerslev, Chou and Kroner, 1992). Schwert (1989) agreed with this argument. Fama (1965) also found the similar evidence. Baillie and Bollerslev (1991) observed that the volatility is predictable in the sense that it is typically higher at the beginning and at the close of trading period. Akgiray (1989) found that GARCH (1, 1) has better explanatory power to predict future volatility in US stock market. Pashakwale and Murinde (2001) modeled volatility in stock markets of Hungary and Poland using daily indexes. They found that GARCH(1,1) accounted for nonlinearity and volatility clustering. Poon and Granger (2003) provided comprehensive review on volatility forecasting. They examined the methodological and empirical findings of 93 research papers and provided synoptic view of the volatility literature on forecasting. They found that ARCH and GARCH classes of time series models are very useful in measuring and forecasting volatility.

In the Indian Context, Roy and Karmakar (1995) focused on the measurement of average level of volatility as the standard deviation in the Indian Stock Market and examined that volatility was highest in the year 1992. Goyal (1995) examined the nature and trend of the stock return volatility in the Indian Stock Market and assessed the impact of carry forward facility on the level of volatility. Reddy (1997) analyzed the establishment of NSE and introduction of BSE online trading (BOLT) on the stock market volatility as sample standard deviation. Kaur (2002) analyzed the extent and pattern of stock market volatility, modeled the volatility during 1990-2000 and examined the effect of company size, FII, day of the week effect on volatility. Ajay Pandey (2002) modeled the volatility of S&P CNX Nifty using different class of estimators and ARCH / GARCH class of models. Balaban, Bayar and Faff (2002) investigated the forecasting performance of both ARCH-type models and non-ARCH models applied to 14 different countries. They observed that non-ARCH models usually produce better forecast than ARCH type models. Finally, Exponential GARCH is the best among ARCH-type models. Pan and Zhang (2006) use Moving Average, Historical Mean, Random Walk, GARCH, GJR-GARCH, EGARCH and APARCH to forecast volatility of two Chinese Stock Market indices; Shanghai and Shenzhen. The study found that Among GARCH models, GJR-GARCH and EGARCH outperforms other ARCH models for Shenzhen stock market.

Magnus and Fosu (2007) employed Random Walk, GARCH(1,1), TGARCH(1,1) and EGARCH(1,1) to forecast Ghana Stock Exchange. GARCH(1,1) provides the best forecast according to three different criterias out of four: On the other hand, EGARCH and Random Walk produces the worst forecast.

Foregoing discussion suggests that the modeling of stock markets volatility and its forecasting is of great importance to academics, policy makers, and financial markets participants. Predicting volatility might enable one to take risk-free decision making including portfolio selection and option pricing. High levels of volatility in a stock market can lead to a general erosion of investors' confidence and an outflow of capital from stock markets, volatility has become a matter of mutual concern for government, management, brokers and investors. It is therefore necessary for us to explore stock market volatility and also identify a model that gives better prediction.

The rest of the paper is organized as follows. Section II provides research design used in the study. Empirical results are discussed in Section III. Section IV summarizes.

Research Design

Period of study
We collected data on daily closing price of Nifty of National
Stock Exchange from January 1, 2010 to June 27, 2014. It consists of 1122 observations. The period of the study is the most recent one. These stock markets have become increasingly integrated. The trades between countries have increased. They are playing an important role in the world economy. These might have influenced the behavior and the pattern of volatility and therefore it will be instructive to analyze volatility in this period.

Methodology

Daily returns are identified as the difference in the natural logarithm of the closing index value for the two consecutive trading days.

Volatility is defined as:

\[
\sigma^2 = \frac{1}{n-1} \sum_{t=1}^{n} (R_t - \bar{R})^2
\]

where \( \bar{R} \) = Average return (logarithmic difference) in the sample.

In comparing the performance of linear model with its nonlinear counterparts, we first used ARIMA models. Nelson (1990b) explains that the specification of mean equation bears a little impact on ARCH models when estimated in continuous time. Several studies recommend that the results can be extended to discrete time. We follow a classical approach of assuming the first order autoregressive structure for conditional mean as follows:

\[
R_t = \alpha_0 + \alpha_1 R_{t-1} + \varepsilon_t
\]

where \( R_t \) is a stock return, \( \alpha_0 + \alpha_1 R_{t-1} \) is a conditional mean and \( \varepsilon_t \) is the error term in period t. The error term is further defined as:

\[
\varepsilon_t = \nu_t \sigma_t
\]

where \( \nu_t \) is white noise process that is independent of past realizations of \( \varepsilon_t \). It has zero mean and standard deviation of one. In the context of Box and Jenkins (1976), the series should be stationary before ARIMA models are used. Therefore, Augmented Dickey Fuller test (ADF) is used to test for stationarity of the return series. It is a test for detecting the presence of stationarity in the series. The early and pioneering work on testing for a unit root in time series was done by Dickey and Fuller (1979 and 1981). If the variables in the regression model are not stationary, then it can be shown that the standard assumptions for asymptotic analysis will not be valid. ADF tests for a unit root in the univariate representation of time series. For a return series \( R_t \), the ADF test consists of a regression of the first difference of the series against the series lagged k times as follows:

\[
\Delta R_t = \alpha + \beta \Delta R_{t-1} + \sum_{i=1}^{p} \beta_i \Delta R_{t-i} + \varepsilon_t
\]

\[
\Delta R_t = R_t - R_{t-1}; R_t = \ln(R_t)
\]

The null hypothesis is \( H_0: \delta = 0 \) and \( H_1: \delta < 1 \). The acceptance of null hypothesis implies nonstationarity. We can transform the nonstationary time series into stationary time series either by differencing or by detrending. The transformation depends upon whether the series is difference stationary or trend stationary.

One needs to specify the form of the second moment, variance, \( \sigma^2_t \) for estimation. ARCH and GARCH models assume conditional heteroscedasticity with homoscedastic unconditional error variance. That is, the changes in variance are a function of the realizations of preceding errors and these changes represent temporary and random departure from a constant unconditional variance. The advantage of GARCH model is that it captures the tendency in financial data for volatility clustering. Therefore, enables us to make the connection between information and volatility explicit since any change in the rate of information arrival to the market will change the volatility in the market. In empirical applications, it is often difficult to estimate models with large number of parameters, say ARCH (q). To circumvent this problem, Bollerslev (1986) proposed GARCH (p, q) model. The conditional variance of the GARCH (p,q) process is specified as:

\[
h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}
\]

with \( \alpha_0 > 0, \alpha_1, \alpha_2, ........ \alpha_q > 0 \) and \( \beta_1, \beta_2, \beta_3, .... \); \( \beta_q > 0 \) to ensure that conditional variance is positive. In GARCH process, unexpected returns of the same magnitude (irrespective of their sign) produce same amount of volatility. The large GARCH lag coefficients \( \beta_i \) mean that shocks to conditional variance takes a long time to die out, so volatility is persistent. Large GARCH error coefficient \( \gamma \) means that volatility reacts quite intensely to market movements and so if \( \gamma \) is relatively high, then volatility tends to be ‘spiky’. If \( \alpha + \beta \) is close to unity, then a shock at time \( t \) will persist for many future periods. A high value of it implies a ‘long memory’.

**EGARCH Model**

GARCH models successfully capture thick tailed returns, and volatility clustering, but they are not well suited to capture the “leverage effect” since the conditional variance is a function only of the magnitudes of the lagged residuals and not their signs.

In the exponential GARCH (EGARCH) model of Nelson (1991) \( \sigma_t^2 \) depends upon the size and the sign of lagged residuals. The specification for the conditional variance is:

\[
\log(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{j=0}^{q} \alpha_j \frac{\varepsilon_{t-j}}{\sigma_{t-j}} + \sum_{h=1}^{q} \gamma_h \frac{\varepsilon_{t-h}}{\sigma_{t-h}}
\]
Note that the left-hand side is the log of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be nonnegative thus eliminating the need for parameter restrictions to impose non-negativity as in the case of ARCH and GARCH models. The presence of leverage effects can be tested by the hypothesis that 
\[
\gamma_h < 0.
\]
The impact is asymmetric if 
\[
\gamma_h \neq 0.
\]

**TGARCH Model**

In ARCH / GARCH models both positive and negative shocks of same magnitude will have exactly same effect in the volatility of the series. T-GARCH model helps in overcoming this restriction. TARCH or Threshold GARCH model was introduced independently by Zakoï (1994) and Glosten, Jagannathan and Runkle (1993). The generalized specification for the conditional variance is given by:

\[
\sigma_t^2 = \alpha_1 \sigma_{t-1}^2 + \alpha_i \varepsilon_{t-1}^2 + \gamma_h \varepsilon_{t-h}^2 d_{t-h}
\]

Equation 7

Where 
\[
d_t = 1 \text{ if } \varepsilon_t < 0 \text{ and zero otherwise.}
\]

In this model, good news, \( \varepsilon_{t-1} > 0 \), and bad news, \( \varepsilon_{t-1} < 0 \), have differential effect on the conditional variance; good news has an impact of \( \alpha_i \) while bad news has an impact of \( \alpha_i + \gamma_i \). If \( \gamma_i > 0 \) bad news increases volatility, and we say that there is a leverage effect for the i-th order. If \( \gamma_i \neq 0 \), the news impact is asymmetric. The main target of this model is to capture asymmetries in terms of positive and negative shocks.

**Forecasting Evaluation**

Root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil inequality coefficient (TIC) are employed to measure the accuracy of the forecasting models.

\[
\text{RMSE} = \sqrt{\frac{\sum_{t=184}^{365} (\sigma_{a,t} - \sigma_{f,t})^2}{182}}
\]

\[
\text{MAE} = \frac{\sum_{t=184}^{365} |\sigma_{a,t} - \sigma_{f,t}|}{182}
\]

\[
\text{MAPE} = 100 \frac{\sum_{t=184}^{365} |\sigma_{a,t} - \sigma_{f,t}|}{\sum_{t=184}^{365} \sigma_{a,t}}
\]

Where \( \sigma_{a,t} \) is the actual volatility \( \sigma_{f,t} \), and is the forecasted volatility.

The model with better forecasting power has lower values of all the above measures compare to other models.

**III. Empirical results**

The descriptive statistics for the return series include mean, standard deviation, skewness, kurtosis, Jarque-Bera and Ljung Box. ARCH-LM statistics are also exhibited in the Table 1.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Nifty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00071</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.01342</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.14196</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>18.95906</td>
</tr>
<tr>
<td>Jarque-Bera Statistics</td>
<td>14706.5(0.000)</td>
</tr>
<tr>
<td>Q(12)</td>
<td>62.96(0.000)</td>
</tr>
<tr>
<td>ARCH LM statistics</td>
<td>1.09(0.289)</td>
</tr>
<tr>
<td>ARCH LM statistics</td>
<td>11.59(0.041)</td>
</tr>
</tbody>
</table>

Notes: ARCH LM statistic is the Lagrange multiplier test statistic for the presence of ARCH effect. Under null hypothesis of no heteroscedasticity, it is distributed as \( \chi^2(k) \). \( Q^2(12) \) is the Ljung Box statistic identifying the presence of autocorrelation in the squared returns. Under the null hypothesis of no autocorrelation, it is distributed as \( \chi^2(k) \).

The mean returns for all the stock indices are very close to zero indicating that the series are mean reverting. The return distribution is negatively skewed, indicating that the distribution is non-symmetric. Large value of Kurtosis suggests that the underlying data are leptokurtic or thick tailed and sharply peaked about the mean when compared with the normal distribution. Since GARCH model can feature this property of leptokurtosis evidence in the data.

The Jarque-Bera statistics calculated and reported in the Table 1 to test the assumption of normality. The results show that the null hypothesis of normality in case of both the stock markets is rejected.

The Ljung-Box LB2 (12) statistical values of all the series re-
respectively rejects significantly the zero correlation null hypothesis. It suggests that there is a clustering of variance. Thus, the distribution of square returns depends on current square returns as well as several periods' square returns, which will result in volatility clustering.

Stationarity condition of the Sensex-daily return series were tested by Augmented Dickey-Fuller Test (ADF). The results of this test are reported in the Table 2.

<p>| Table 2 |
|------------------|------------------|</p>
<table>
<thead>
<tr>
<th>Level</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF statistics in level series shows presence of unit root in the stock market as its probability value is greater than 0.05. It suggests that the price series is nonstationary. It is, therefore, necessary to transform the series to make it stationary by taking its first difference. ADF statistics reported in the Table 2 show that the null hypothesis of a unit root is rejected. The computed values for the index is statistically significant. Thus, the result shows that the first difference series is stationary. To test for heteroscedasticity, the ARCH-LM test is applied to the series. The results are reported in Table 1. The ARCH-LM test at lag length 1 and 5 indicate presence of ARCH effect in the residuals in both the stock markets. It implies clustering of volatility where large changes tend to be followed by large changes, of either sign and small changes tend to be followed by small changes (Engle, 1982 and Bollerslev, 1986). The Conditional volatility of returns may not only be dependent on the magnitude of error terms but also on its sign. We checked for asymmetry in both the stock markets using EGARCH and TARCH models. The results are reported in Table 3.</td>
<td>AIC</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>GARCH(1,1)</td>
<td>0.0000(0.000)</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>-0.5952(0.000)</td>
</tr>
<tr>
<td>TARCH(1,1)</td>
<td>0.0396(0.000)</td>
</tr>
<tr>
<td>Coefficients of Asymmetric Models</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.0000(0.000)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.0000(0.000)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.0000(0.000)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0000(0.000)</td>
</tr>
<tr>
<td>$\alpha_1 + \beta_1$</td>
<td>0.5052</td>
</tr>
<tr>
<td>TARCH(1,1)</td>
<td>0.0000(0.000)</td>
</tr>
</tbody>
</table>

The above findings indicate that there is no ARCH effect left after estimating the models because the results of F-statistics or ARCH-LM test after fitting the model are statistically insignificant as its probability value is higher than 0.05. It, therefore, suggests that the estimated models are better fit. Conditional volatility of returns may not only be dependent on the magnitude of error terms but also on its sign. We checked for asymmetry in both the stock markets using EGARCH and TARCH models. The results are presented in the Table 3.

The analysis of this EGARCH model suggests that its coefficient (-0.0908) is significant, the leverage effect term $\gamma$ is negative and statistically different from zero, indicating the existence of leverage effect in the stock market returns during the sample period.

Similarly, results of TARCH model estimation are listed in Table 3. Most importantly, the leverage term ($\gamma$), represented by $(RESID(-1)\gamma RESID(-1)=0)$ is here greater than zero and highly significant. Its value is 0.1262. This reinforces the assumption that negative and positive shocks have different impact on the volatility of daily returns. Here good news has an impact of $\gamma = 0.0396$, while the bad news has an impact of $\alpha_1 + \gamma$ which is equal to 0.1658. Thus, it can be said that negative or bad news creates greater volatility than positive or good news in both the stock markets.

The model selection criterion AIC and SBC reported in Table 3 select GARCH(1,1) models as their values are smallest for GARCH(1,1) models. Now, we evaluate the models on the basis of their forecasting accuracy. The results are reported in Table 4.

| Table 4 |
|------------------|------------------|
| Model | RMSE | MAE | MAPE | TIC |
|------------------|------------------|
| GARCH(1,1) | 0.000125 | 0.000094 | 0.000094 |
| EGARCH(1,1) | 0.000129 | 0.000101 | 0.000101 |
| GJR-GARCH(1,1) | 0.000132 | 0.000129 | 0.000129 |

Table 4 gives the actual forecast error statistics for each model. In the case of RMSE, MAE and MAPE, GARCH provides the best volatility forecast. The Theil Inequality Coefficient (TIC) is a scale invariant measure that always lies between Zero and one, where Zero indicates a perfect fit. Looking at this coefficient we can say that GJR-GARCH(1,1) model is the best forecasting model. All the forecasting measures hints at GARCH(1,1) model for better forecasting of conditional volatility.

IV. Summary
The volatility in the Nifty exhibits the persistence of volatility, mean reverting behavior and volatility clustering. Various diagnostic tests indicate volatility clustering and the response to news arrival is asymmetrical, meaning that impact of good and bad news is not the same. By the application of asymmetrical GARCH models like EGARCH and TARCH, we conclude that there is a presence of leverage effect in both the stock markets in India. These models suggest that the volatility appears to be more when price decline than when price increases.

We evaluated the models on the basis of model selection cri-
terion and their forecasting accuracy. We used AIC and SBC criteria to select best fitting model and RMSE, MAE, MAPE and TIC to check their forecasting accuracy. Our results indicate that GARCH (1, 1) is the best forecasting model.

References


